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Response of the beams on random Pasternak foundations subjected to harmonic moving loads †

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Abstract

Dynamic response of infinite beams supported by random viscoelastic Pasternak foundation subjected to harmonic moving loads is studied. Vertical stiffness in the support is assumed to follow a stochastic homogeneous field consisting of a small random variation around a deterministic mean value. By employing the first order perturbation theory and calculating appropriate Green's functions, the variance of the deflection and bending moment are obtained analytically in integral forms. To simulate the induced uncertainty, two practical cases of cosine and exponential covariance are utilized. A frequency analysis is performed and influences of the correlation length of the stiffness variation on the beam responses are investigated. It is found that in each frequency response there is a peak value of frequency, which behaves as a decreasing function of the correlation length. Among two coefficient of variation of the beam deflection and the bending moment, the former is higher in the case of exponential covariance and it is independent of the magnification of the correlation length.

***Keywords*:** Timoshenko beam; Moving load; Random vibration; Viscoelastic foundation.

1. Introduction

The investigation on the consequences of a force moving along an infinite beam rested on viscoelastic foundation is of great theoretical and practical significance, in particular its application in the modeling of railway track [1, 2]. In most of the cases and for simplicity, the track parameters have been assumed to have a deterministic characteristic equation, but recently mechanical properties of the ballast have been found to vary significantly along the track. The uncertainty of the track parameters and surface irregularity of the rail are two important sources of random vibration in railway tracks and they lead to vibrations of the track even if no other external excitation is applied. Many reasons such as non-uniformity in packing the ballast and also some natural and operational sources can easily cause random distribution of the track properties along its length. Because of this random distribution, coefficients of the differential equations of motion become random functions of the position and as the moving load is travelling on the foundation, the randomness of the foundation parameters causes an excessive vibration. Since this excessive vibration arises from an internal source it is called 'parametric excitation'. In the literature, a few studies have been carried out and some related problems have been treated numerically and analytically in recent years. Dynamic response of an Euler-Bernoulli beam resting on a Winkler random foundation has been obtained analytically in [3] under an idealistic assumption of the white noise spectral density for the uncertainty of the foundation stiffness. Behavior of an infinite Euler-Bernoulli beam on a Kelvin foundation

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with randomly varying parameters along the beam is examined in [4]. The authors used finite element method and found that randomness of the foundation stiffness is of greater importance than the uncertainty in the damping. They also studied the influence of the load speed on the beam responses. Interaction of an imperfect track and a moving random load using the method of integral spectral decomposition was studied in [5, 6] studied. They showed that imperfections could not be modeled by white noise. The response of a single-degree-of-freedom vehicle moving uniformly along an Euler-Bernoulli beam resting on a modified Kelvin foundation with randomly distributed stiffness has been studied in [7, 8]. They showed that the vertical motion of a moving vehicle and the beam deflection increase with the vehicle speed and also with the correlation length of the stiffness variation. A comprehensive investigation on the literature indicates that there are few works published in this area and lack of a frequency analysis is quite clear. Vibration of infinite beams on various type of supports consisting of linear, nonlinear and uncertain viscoelastic foundations subjected to harmonic moving loads was studied in [9-12].

In the present paper, using the first order perturbation method, the response analysis of an infinite Timoshenko beam on the viscoelastic foundation under a harmonic moving load is carried out. To simulate the behavior of the foundation, Pasternak viscoelastic model is used. This model includes a Kelvin foundation in conjunction with a shear elastic layer which can model the track more realistically [7, 8, 13]. Based on this model, the coefficient of variation of the beam deflection and bending moment at the point of the application of the moving force are obtained. Using the complex Fourier transformation, appropriate Green's functions are presented, and the mean and variance parts of the response of the beam are calculated analytically in the integral forms. Using the residue theorem, for two practical cases of random foundations, a parametric study is carried out and the effects of correlation length on the coefficient of variations are studied. A frequency analysis is performed as well. The solution and also the parametric study are directed to make the outcomes of this paper more applicable in different branches of railway engineering such as noise analysis of tracks, passenger comfort analysis of railway vehicles, dynamic and fatigue design of tracks in which the frequency response has an important role.

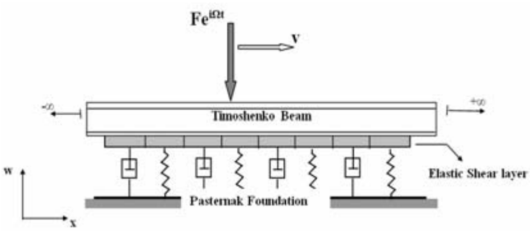

Fig. 1. Timoshenko beam on random Pasternak foundation.

2. Theory and formulation

Fig. 1 illustrates a beam on a random viscoelastic foundation under a harmonic moving load. By using Hamilton's principle and employing the Timoshenko beam theory, one can obtain the differential equations of the motion as ([9-12])

$$\rho A\frac{\partial^2 w(x,t)}{\partial t^2}+k*AG\left(\frac{\partial\phi(x,t)}{\partial x}-\frac{\partial^2 w(x,t)}{\partial x^2}\right)+P_f(x,t)=-Fe^{i\Omega t}\delta(x-vt) \quad (1)$$

$$EI\frac{\partial^2\phi(x,t)}{\partial x^2}-k*AG\left(\phi(x,t)-\frac{\partial w(x,t)}{\partial x}\right)=\rho I\frac{\partial^2\phi(x,t)}{\partial t^2} \quad (2)$$

in which A, E, G, I, k* and ρ are cross-sectional area of the beam, the modulus of elasticity, shear modulus, second moment of area, sectional shear coefficient and beam material density and v, Ω, w, and ϕ are the load speed and frequency, beam deflection and beam slope due to bending, respectively. Moreover, Pf represents the force induced by the foundation per unit length of the beam. In the case of a random viscoelastic foundation it can be calculated as:

$$P_f(x,t)=k(x)w(x,t)-\mu\frac{\partial^2 w(x,t)}{\partial x^2}+c\frac{\partial w(x,t)}{\partial t} \quad (3)$$

in which, k (x) and μ are the foundation stiffness and the shear layer stiffness and c is damping coefficient of the foundation. The foundation stiffness can be decomposed into its constant mean value, km, and corresponding stochastic component, *k* (*x*):

$$k(x)=k_m+\varepsilon k_s^*(x)=k_m+k_s(x) \quad (4)$$

in which k_s* (x) is a random stationary ergodic function with zero mean value and ε is the small parameter. Using the regular perturbation method one can assume [11, 14]:

$$w(x,t) = w_0(x,t) + \varepsilon w_1(x,t) + \varepsilon^2 w_2(x,t) + ... \tag{5}$$

$$\phi(x,t) = \phi_0(x,t) + \varepsilon \phi_1(x,t) + \varepsilon^2 \phi_2(x,t) + ... \tag{6}$$

Substituting Eqs. (5) and (6) in Eqs. (1) and (2) and comparing the terms with the same powers of parameter ε, one can get:

$$\begin{aligned}
&\rho A \frac{\partial^2 w_0(x,t)}{\partial t^2} + k_m w_0(x,t) + k*AG\left(\frac{\partial \phi_0(x,t)}{\partial x} - \frac{\partial^2 w_0(x,t)}{\partial x^2}\right) - \mu \frac{\partial^2 w_0(x,t)}{\partial x^2} + c\frac{\partial w_0(x,t)}{\partial t} = -Fe^{i\Omega t}\delta(x-vt) \\
&EI \frac{\partial^2 \phi_0(x,t)}{\partial x^2} - k*AG\left(\phi_0(x,t) - \frac{\partial w_0(x,t)}{\partial x}\right) = \rho I \frac{\partial^2 \phi_0(x,t)}{\partial t^2}
\end{aligned} \tag{7}$$

$$\begin{aligned}
&\rho A \frac{\partial^2 w_1(x,t)}{\partial t^2} + k_m w_1(x,t) + k*AG\left(\frac{\partial \phi_1(x,t)}{\partial x} - \frac{\partial^2 w_1(x,t)}{\partial x^2}\right) - \mu \frac{\partial^2 w_1(x,t)}{\partial x^2} + c\frac{\partial w_1(x,t)}{\partial t}) = -k_s^*(x,t) w_0(x,t) \\
&EI \frac{\partial^2 \phi_1(x,t)}{\partial x^2} - k*AG\left(\phi_1(x,t) - \frac{\partial w_1(x,t)}{\partial x}\right) = \rho I \frac{\partial^2 \phi_1(x,t)}{\partial t^2}
\end{aligned} \tag{8}$$

To calculate the steady state response of the beam we use the Galilean co-ordinate transformation such as:

$$s = x - vt \tag{9}$$

Since the beam length is considered to be infinite, the boundary conditions are

$$\begin{cases}
\lim\limits_{s\to\pm\infty} w_{st}(s) = \lim\limits_{s\to\pm\infty} \dfrac{dw_{st}(s)}{ds} = \lim\limits_{s\to\pm\infty} \dfrac{d^2 w_{st}(s)}{ds^2} = 0 \\
\lim\limits_{s\to\pm\infty} \phi_{st}(s) = \lim\limits_{s\to\pm\infty} \dfrac{d\phi_{st}(s)}{ds} = 0
\end{cases} \tag{10}$$

in which w_{st} (s) and φ_{st} (s) are steady state descriptions of w (x,t) and φ (x,t) and consequently can be written as

$$w_{st}(s) = w_{0-st}(s) + w_{1-st}(s)\varepsilon + w_{2-st}(s)\varepsilon^2 + ... \tag{11}$$

$$\phi_{st}(s) = \phi_{0-st}(s) + \phi_{1-st}(s)\varepsilon + \phi_{2-st}(s)\varepsilon^2 + ... \tag{12}$$

The above boundary conditions represent the fact that at points far enough from the point of the load application the central line deflection, its slope, curvature, shear force and bending moment are all approaching to zero.

According to the Timoshenko beam theory, the bending moment along the beam is given by

$$M_{st}(s) = EI\frac{\partial \phi_{st}(s)}{\partial s} = M_{0-st}(s) + M_{1-st}(s) + M_{2-st}\,\varepsilon(s)\varepsilon^2 + ... \tag{13}$$

Using the state variable transformation of

$$\begin{cases}
w_{j-St}(s) = w_j(s)\, e^{i\Omega t}\,;\; w_j(s) = w_{jA}(s) e^{i\psi_j(s)} \\
\phi_{j-st}(s) = \phi_j(s)\, e^{i\Omega t}\,;\; \phi_j(s) = \phi_{jA}(s) e^{i\theta_j(s)}
\end{cases} \tag{14}$$

In which, for j=0,1,…, w_{jA} and ψ_j are the amplitude and the phase of the steady state response, w_j and similarly ϕ_{jA} and θ_j are the amplitude and the phase of φ_j. After utilizing the chain rule differentiation on the Eqs. (7) and (8) and considering Eq. (14), one can get

$$\begin{cases}
\rho\left(v^2 \dfrac{d^2}{ds^2} - 2i\Omega v \dfrac{d}{ds} - \Omega^2\right) w_0(s) + k*AG\left(\dfrac{d\phi_0(s)}{ds} - \dfrac{d^2 w_0(s)}{ds^2}\right) - \mu \dfrac{d^2 w_0(s)}{ds^2} + c\left(-v\dfrac{d}{ds} + i\Omega\right) w_0(s) + k_m w_0(s) = -F\delta(s) \\
EI\dfrac{d^2\phi_0(s)}{ds^2} - k*AG\left(\phi_0(s) - \dfrac{dw_0(s)}{ds}\right) = \rho I\left(v^2 \dfrac{d^2}{ds^2} - 2i\Omega v \dfrac{d}{ds} - \Omega^2\right)\phi_0(s)
\end{cases} \tag{15}$$

$$\begin{cases} \rho(v^2\frac{d^2}{ds^2}-2i\Omega v\frac{d}{ds}-\Omega^2)w_1(s)+ \\ k*AG(\frac{d\phi_1(s)}{ds}-\frac{d^2w_1(s)}{ds^2})-\mu\frac{d^2w_1(s)}{ds^2}+ \\ c(-v\frac{d}{ds}+i\Omega)w_1(s)+k_m w_1(s)=-k_s^*(s)w_0(s) \\ EI\frac{d^2\phi_1(s)}{ds^2}-k*AG(\phi_1(s)-\frac{dw_1(s)}{ds})= \\ \rho I(v^2\frac{d^2}{ds^2}-2i\Omega v\frac{d}{ds}-\Omega^2)\phi_1(s) \end{cases} \quad (16)$$

After applying the following complex Fourier transformation, i.e., Eq. (17) to Eq. (15) and imposing the boundary conditions of Eq. (10), Fourier transforms of w_0 (s) and ϕ_0 (s) i.e. W_0 (q) and Φ_0 (q) are obtained as:

$$F(q)=\int_{-\infty}^{+\infty} f(s)\, e^{-isq}\, ds \quad (17)$$

$$W_0(q)=\frac{-(B_{1-0}q^2+B_{2-0}q+B_{3-0})F}{B_{4-0}q^4+B_{5-0}q^3+B_{6-0}q^2+B_{7-0}q+B_{8-0}} \quad (18)$$

$$\Phi_0(q)=\frac{(B_{9-0}q)F}{B_{4-0}q^4+B_{5-0}q^3+B_{6-0}q^2+B_{7-0}q+B_{8-0}} \quad (19)$$

General definitions of all the above coefficients are listed in Table 1. If an inverse Fourier transform is taken from both sides of Eqs. (18) and (19) one obtains:

$$w_0(s)=\frac{F}{2\pi}\int_{-\infty}^{+\infty}\frac{-(B_1q^2+B_2q+B_3)e^{isq}\,dq}{B_4q^4+B_5q^3+B_6q^2+B_7q+B_8} \quad (20)$$

$$M_0(s)=\frac{F}{2\pi}\int_{-\infty}^{+\infty}\frac{(B_{10}q^2)e^{isq}dq}{B_4q^4+B_5q^3+B_6q^2+B_7q+B_8} \quad (21)$$

Using a similar procedure for Eq. (16) and using appropriate Green's functions and the convolution integral theorem, the closed form solutions are obtained for j=1,2,… as

$$\begin{cases} w_j(s)=-\int_{-\infty}^{+\infty} k_s^*(\xi)w_{j-1}(\xi)G_w(s-\xi)\,d\xi \\ M_j(s)=-\int_{-\infty}^{+\infty} k_s^*(\xi)w_{j-1}(\xi)G_M(s-\xi)\,d\xi \end{cases} \quad (22)$$

in which Green's functions G_w (ξ) and G_M (ξ) are defined as

$$G_w(\xi)=\frac{1}{2\pi}\int_{-\infty}^{+\infty}\frac{(B_1q^2+B_2q+B_3)e^{i\xi q}\,dq}{B_4q^4+B_5q^3+B_6q^2+B_7q^1+B_8} \quad (23)$$

$$G_M(\xi)=\frac{1}{2\pi}\int_{-\infty}^{+\infty}\frac{B_{10}q^2\, e^{i\xi q}\,dq}{B_4q^4+B_5q^3+B_6q^2+B_7q^1+B_8} \quad (24)$$

All B_i coefficients in the above Green's functions are defined in Table 1.

It should be noted that the residue theorem is used for calculating the Green's function and also the numerical values of the dynamic response. In this way G_w can be calculated as

$$G_w(\xi)=\pi i\sum_{jr=1}^{n_r}[\mathrm{Res}(Integrand\ \ (G_w(\xi)))]_{z=z_{jr}} \quad (25)$$
$$+2\pi i\sum_{j=1}^{n}[\mathrm{Res}(Integrand\ \ (G_w(\xi)))]_{z=z_j}\quad for\ \xi\ge 0$$

$$G_w(\xi)=-\pi i\sum_{jr=1}^{n_r}[\mathrm{Res}(Integrand\ \ (G_w(\xi)))]_{z=z_{jr}} \quad (26)$$
$$-2\pi i\sum_{k=1}^{n}[\mathrm{Res}(Integrand\ \ (G_w(\xi)))]_{z=z_k}\quad for\ \xi< 0$$

in which, z_j represents the poles of the integrand of G_w in the upper half and z_k represents its poles in the lower half of the complex plane and z_{jr} represents its real poles. Moreover, the G_M is obtained in a similar trend.

From Eqs. (11) and (12), the mean values of the beam deflection and bending moment are:

Table 1. Definition of the coefficients appeared in related Green's functions.

Parameter	Definition
B_1	$EI-\rho Iv^2$
B_2	$2\Omega v\rho I$
B_3	$k*AG-\rho I\Omega^2$
B_4	$(EI-\rho Iv^2)(k*AG-\rho Iv^2+\mu)$
B_5	$EI(2\Omega v\rho A-civ)+2\Omega v\rho I(k*AG+\mu)$ $+\rho I^2v^2c-4\Omega v^3\rho^2 IA$
B_6	$\rho Av^2(\rho I\Omega^2-k*AG)+$ $2\Omega\ v\rho I(2\Omega\ v\rho A-civ)+$ $(k+ci\Omega\ -\Omega^2\rho A)(EI-\rho Iv^2)+$ $\mu(k*AG-\rho I\Omega^2)-k*AG\rho I\Omega^2$
B_7	$(2\Omega v\rho A-civ)(k*AG-\rho I\Omega^2)$ $+2\Omega\ v\rho I(k+ci\Omega\ -\Omega^2\rho A)$
B_8	$(k+ci\Omega-\Omega^2\rho A)(k*AG-\rho I\Omega^2)$
B_9	$k*AG\,i$
B_{10}	$-k*AG\,EI$

$$E[w_{st}(s)] = E[w_{0-st}(s)] + E[w_{1-st}(s)]\varepsilon + E[w_{2-st}(s)]\varepsilon^2 + ... \tag{27}$$

$$E[M_{st}(s)] = E[M_{0-st}(s)] + E[M_{1-st}(s)]\varepsilon + E[M_{2-st}(s)]\varepsilon^2 + ... \tag{28}$$

Since the mean value of the stochastic part of the stiffness is zero, Eq. (22) implies that the mean values of the higher perturbation terms are all zero, i.e.,

$$E[w_{j-st}(s)] = 0 \quad j = 1,2,... \tag{29}$$

$$E[M_{j-st}(s)] = 0 \quad j = 1,2,... \tag{30}$$

so

$$E[w_{st}(s)] = E[w_{0-st}(s)] \tag{31}$$

$$E[M_{st}(s)] = E[M_{0-st}(s)] \tag{32}$$

The centered values of the deflection and bending moment are:

$$\hat{w}(s) = w_{st}(s) - E[w_{st}(s)] \approx w_{1-st}(s)\varepsilon \tag{33}$$

$$\hat{M}(s) = M_{st}(s) - E[M_{st}(s)] \approx M_{1-st}(s)\varepsilon \tag{34}$$

With regard to the definition of the covariance of a random function [15] and Eq. (20-22), the covariance of the deflection and the bending moment are obtained as

$$Cov_{ww}(s_1,s_2) = E[\varepsilon w_{1-st}(s_1)\varepsilon w_{1-st}(s_2)] = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} w_0(\xi_1)w_0(\xi_2)G_w(s_1-\xi_1)G_w(s_2-\xi_2)Cov_{kk}(\xi_1,\xi_2)d\xi_1\xi_2 \tag{35}$$

$$Cov_{MM}(s_1,s_2) = E[\varepsilon M_{1-st}(s_1)\varepsilon M_{1-st}(s_2)] = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} M_0(\xi_1)M_0(\xi_2)G_M(s_1-\xi_1)G_M(s_2-\xi_2) Cov_{kk}(\xi_1,\xi_2)d\xi_1\xi_2 \tag{36}$$

in which $Cov_{kk}(\xi_1, \xi_2)$ is the covariance of the foundation stiffness. Then, the variance of the deflection and the bending moment are

$$\sigma_w^2(s) = Cov_{ww}(s,s) \tag{37}$$

$$\sigma_M^2(s) = Cov_{MM}(s,s) \tag{38}$$

3. Numerical result

For two practical types of foundation stiffness expressed in the form of exponential and cosine covariance functions, a parametric study is carried out and the results are discussed in this section. These covariance functions are in the form of:

$$Cov_{kk}(\xi_1,\xi_2) = \sigma^2 \exp(-\frac{|\xi_1-\xi_2|}{D}) \tag{39}$$

$$Cov_{kk}(\xi_1,\xi_2) = \sigma^2 \cos(2\pi\frac{\xi_1-\xi_2}{D}) \tag{40}$$

where σ is the standard deviation of the random stiffness and D is the correlation length.

The numerical results are related to the coefficients of variation for the deflection ($C_w(s)$) and the bending moment ($C_M(s)$) at the point of application of the moving force that is (s=0)

$$C_w(0) = \left.\frac{\sigma_w(s)}{E[w(s)]}\right|_{s=0} \tag{41}$$

$$C_M(0) = \left.\frac{\sigma_M(s)}{E[M(s)]}\right|_{s=0} \tag{42}$$

Before the main parametric study and in order to verify the numerical procedure, a special case of white noise is assumed for the covariance of the stiffness. The numerical simulation is carried out in the vicinity of the resonance (f=291 Hz) for the non-damped foundation. Analytical results for the Euler-Bernoulli beam subjected to the harmonic moving point load was obtained in [3]. The obtained results for the Timoshenko beam are compared with those found in [3] in Fig. 2. As illustrated, very good correlation can be seen between the two approaches.

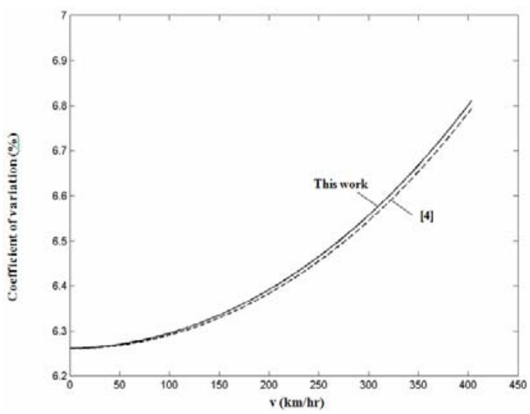

Fig. 2. Coefficient of variation versus the load speed in the vicinity of resonance (special case of the white noise has been assumed for covariance of the stiffness).

Table 2. Properties of the UIC60 rail, track and load [9, 10, 16].

Item	Notation	Value
Rail (UIC60)		
Young's modulus (steel)	E	210 GPa
Shear modulus (steel)	G	77 GPa
Mass density	ρ	7850 kg/m^3
Cross sectional area	A	7.69×10^{-3} m^2
Second moment of area	I	30.55×10^{-6} m^4
Shear coefficient	k*	0.40
Foundation		
Mean stiffness	k_m	202.66 MN/m^2
Viscous damping	c	141.165 kNs/m^2
Stiffness variance	σ^2	10.822×10^{14} N^2/m^4
Moving Load		
Load	F	65 kN

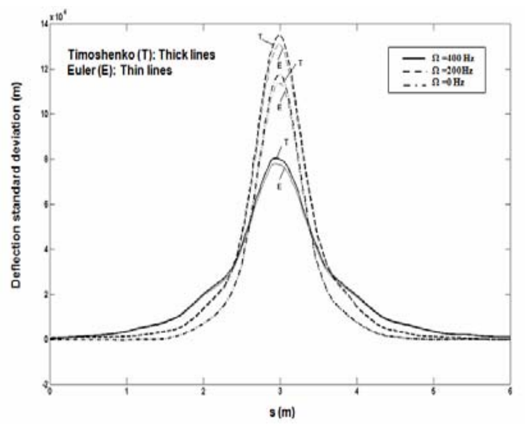

Fig. 3. Frequency effect on the distribution of the standard deviation of the deflection (v=200 km/hr) (Cosine covariance).

For a real track [17], a frequency analysis is carried out using the above two prescribed covariance functions. The physical and geometrical properties of the track are listed in Table 2. The effects of the load frequency on the distribution of the standard deviation of the beam deflection and bending moment are illustrated in Figs 3-6 for the two abovementioned cases of cosine and exponential covariance. As can be seen, by increasing the load frequency the maximum standard deviation increases up to a point and then decreases. Moreover, it makes the distribution wider along the beam and causes the position of the peak values moves farther back with respect to the point of application of the moving load.

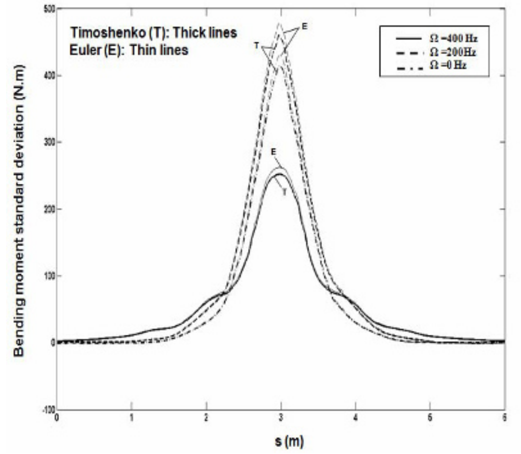

Fig. 4. Frequency effect on the distribution of the standard deviation of the bending moment (v=200 km/hr) (Cosine covariance).

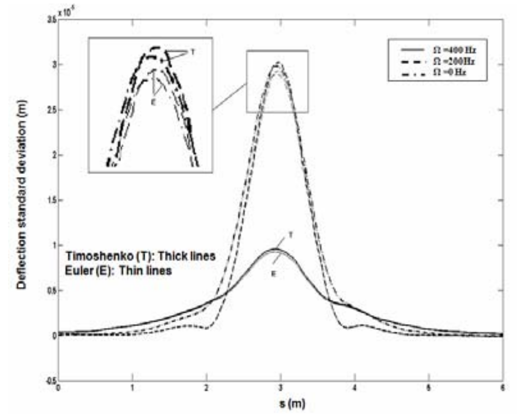

Fig. 5. Frequency effect on the distribution of the standard deviation of the deflection (v=200 km/hr) (Exponential covariance).

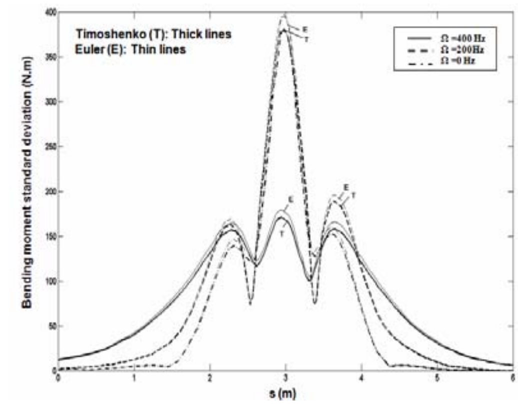

Fig. 6. Frequency effect on the distribution of the standard deviation of the bending moment (v=200 km/hr) (Exponential covariance).

To compare two theories of the Timoshenko and Euler beam, the same procedure has been done for the Euler beam with the governing equation of

$$\rho A \frac{\partial^2 w(x,t)}{\partial t^2} + EI \frac{\partial^4 w(x,t)}{\partial x^4} + k\, w(x,t) + C \frac{\partial w(x,t)}{\partial t} - \mu \frac{\partial^2 w(x,t)}{\partial x^2} = -Fe^{i\Omega t} \delta(x - vt) \quad (43)$$

Numerical results for the Euler beam are also included in Figs 3-6. As seen, one can generally say that the magnitude of the maximum deflection of a Timoshenko beam is larger than that of an Euler beam, and inversely the magnitude of the maximum bending moment of a Timoshenko beam is lower than that of an Euler beam. Having a viewpoint focused on the energy transfer, one can conclude that for a track modeled by a Timoshenko beam on Pasternak foundation larger fraction of the input.

The frequency responses of the coefficient of variation of the deflection and bending moment are illustrated in Figs. 7 and 8. There are two frequencies at which C_w (0) and C_M (0) are at their highest level similar to the frequency response of an S.D.F system. The peak frequency of the coefficient of variation of the deflection is higher than the bending moment and it decreases with increasing of correlation length as well as the coefficient of variation of the bending moment. In the case of exponential covariance, the correlation length has much lower effect on the frequency responses (Figs. 9 and 10). Also can be seen, for the case of exponential covariance the coefficient of variance of the deflection is higher than that of the

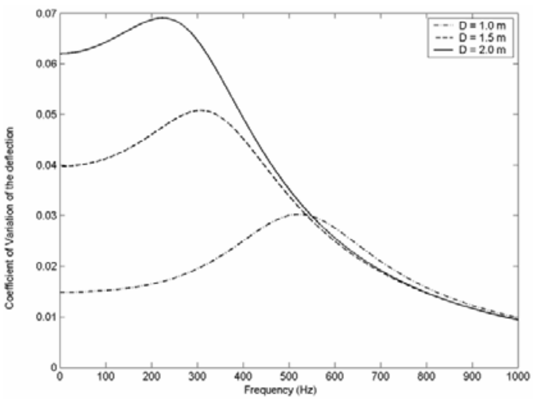

Fig. 7. Frequency response of the beam deflection at s=0 for various correlation lengths (D) (v=200 km/hr) (Cosine covariance).

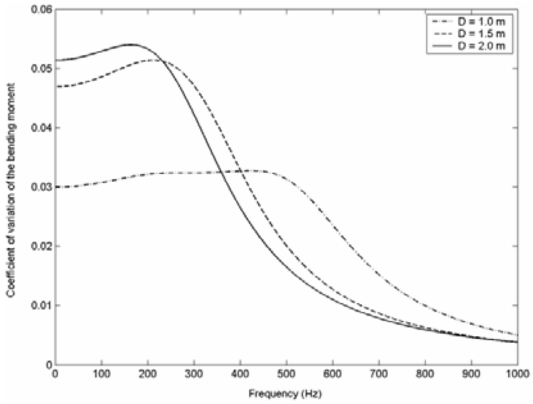

Fig. 8. Frequency response of the beam bending moment at s=0 for various correlation lengths (D) (v=200 km/hr) (Cosine covariance).

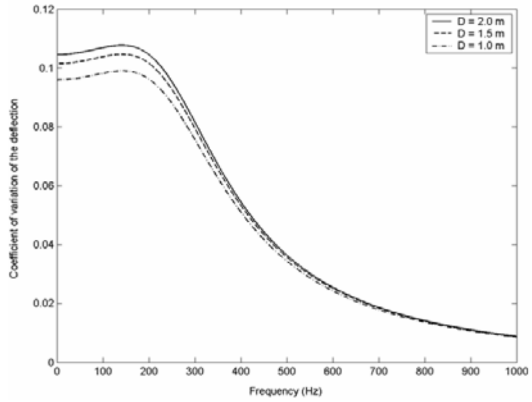

Fig. 9. Frequency response of the beam deflection at s=0 for various correlation lengths (D) (v=200 km/hr) (Exponential covariance).

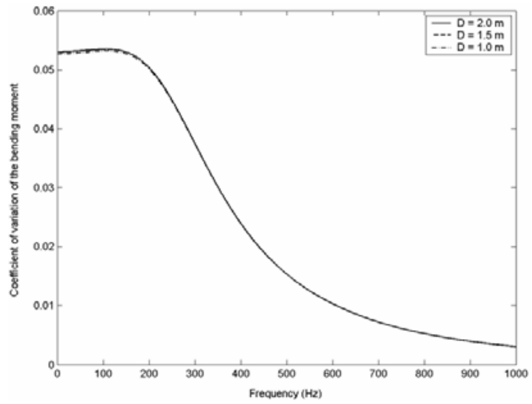

Fig. 10. Frequency response of the beam bending moment at s=0 for various correlation lengths (D) (v=200 km/hr) (Exponential covariance).

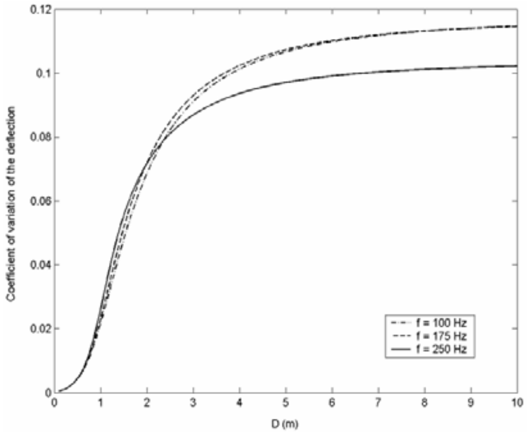

Fig. 11. Relationship between the coefficient of variation of the deflection at s=0 and the correlation length (v=200 km/hr) (Cosine covariance).

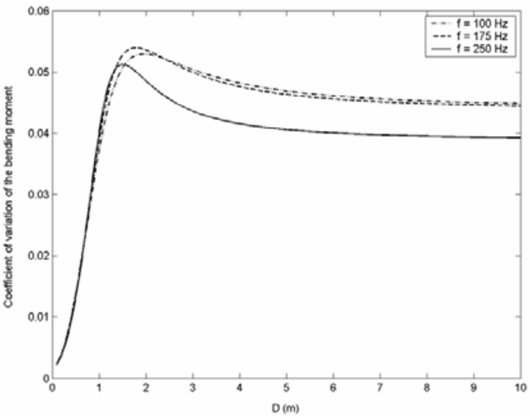

Fig. 12. Relationship between the coefficient of variation of the bending moment at s=0 and the correlation length (v=200 km/hr) (Cosine covariance).

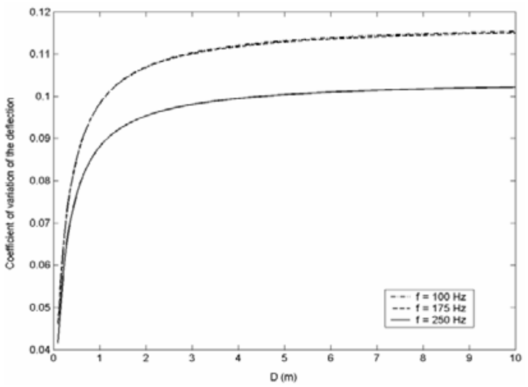

Fig. 13. Relationship between the coefficient of variation of the deflection at s=0 and the correlation length (v=200 km/hr) (Exponential covariance).

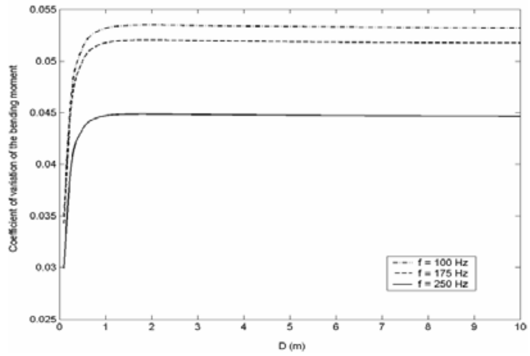

Fig. 14. Relationship between the coefficient of variation of the bending moment at s=0 and the correlation length (v=200 km/hr) (Exponential covariance).

bending moment, but in the case of cosine covariance it depends on the magnification of the correlation length. For the case of cosine covariance the influence of the correlation length on the C_w (0) and C_M (0) is illustrated in Figs. 11 and 12 for three different values of frequencies around the peak frequency. Saturation phenomenon that is getting independence from correlation length at high values of D, can be seen for both C_w (0) and C_M (0). In the case of C_M (0), there is a peak value for D, which is a decreasing function of the frequency value and has a value between 1 to 2 meters. Similar results can be seen for the case of exponential covariance in Figs 13 and 14. In this case there is no peak value of D and also the saturation phenomenon is taking place at much lower values of correlation length.

Energy is transferred to the foundation rather than the beam because of the larger deflections. But in the case of Euler beam on the Pasternak foundation, due to larger values of the bending stresses, larger fraction of the input energy is transferred into the beam (against the Timoshenko beam theory). On the other hand, Euler theory has safer margins for designing of the beam, and Timoshenko theory has safer margins for designing of the foundation.

To give more physical explanations and applications of the numerical results, it should be noted that the obtained results may be easily used for real tracks subjected to moving trains. Since all of the terms of the response (w_j) are linear functions with respect to the magnitude of the moving load, i.e., F (Eq. (20)), the results can be easily extended for the number of moving forces. It means that the superposition principle here is still acceptable and therefore successive moving loads which well model the train loading condition can also be simulated with the same approach.

The same comparison was performed for the coefficient of variation of the two beam theories, Timoshenko and Euler beam. Since the coefficient of variation is a dimensionless parameter and is defined by Eqs. (41-42), the type of the selected theory has no significant effect on the coefficient of variation. This is because any change of the theory leads to the same magnitude of variations both in denominator and numerator of the Eqs. (41) and (42).

4. Conclusions

The response of an infinite beam supported by Pasternak foundation with randomly distributed stiffness subjected to harmonic moving loads was investigated. The mean value and variance of the beam deflection and bending moment were calculated by using the first order perturbation method. Two practical cases of cosine and exponential covariance were used to simulate the uncertainty of the foundation stiffness and a frequency analysis was carried out for the first time in this paper. The influences of the correlation length on the responses were studied as well. Conclusions are as follows:

(1) Using Green's function approach, a closed-form solution in integral form applicable in the frequency analysis of tracks was obtained. The presented method and also the results are applicable in some related fields in railway engineering such as noise analysis of tracks, ride comfort of railway vehicles and also dynamic and fatigue design of railway tracks.

(2) Using the solution method presented, the distribution of the standard deviation of the deflection and bending moment along the beam were calculated and the influences of the load frequency on both of them were investigated. Increasing the load frequency causes the position of the maximum values to move farther back with respect to the load position and also makes them to be distributed in a wider area along the beam.

(3) The parametric study showed that in each frequency response there is a peak value for the frequency which depends on the correlation length. The peak frequency is a decreasing function of the correlation length. Dependency of the peak frequency on the correlation length is based on the covariance type of the foundation stiffness and for the case of cosine covariance it is much stronger than exponential covariance. Moreover, it has a higher value for the deflection than the bending moment.

(4) For the case of exponential covariance, a comparison between two dimensionless parameters of the coefficient of variation of the beam deflection and bending moment showed that the former one is higher. Regarding the unavailability of a device for direct measurement of the bending moment, this result does have a significant advantage in design of railway tracks because it shows that using a typical measured coefficient of variation of the beam deflection is a safe criterion in design of tracks based on the beam's strength.

(5) With increasing the correlation length, the coefficient of variation of the beam deflection and bending moment remain constant; this phenomenon, i.e., saturation, does occur for the two selected cases of exponential and cosine covariance of the stiffness. For the exponential covariance, the saturation occurs in lower correlation lengths other than the cosine covariance. In the latter case, there is a peak value of correlation length between 1 to 2 meters at which coefficient of variation of the bending moment is at its highest level and is a decreasing function of the load frequency.

(6) Numerical results obtained for the two theories of Euler and Timoshenko beams were compared. It was found that the magnitude of the maximum deflection of a Timoshenko beam is larger than that of an Euler beam and inversely the magnitude of the maximum bending moment of a Timoshenko beam is lower with respect to the Euler beam. One can conclude that Timoshenko and Euler theories provide safer margins, respectively, for design of the foundation and the beam.

Nomenclature

A : Cross-sectional area
C : Foundation damping
Cov : Covariance
D : Correlation length
E : Modulus of elasticity
F : Moving force
G : Shear modulus
I : Cross-sectional moment of inertia
k : Foundation stiffness
k* : Sectional shear coefficient of the beam
k_m : Mean stiffness
M : Bending moment

P_f : Foundation force per unit length
s : Distance from the moving load
t : Time
v : Load speed
w : Beam deflection
x : Longitudinal position
δ : Dirac Delta function
μ : Shear layer stiffness
ρ : Beam material density
σ : Standard deviation
ϕ : Beam slope due to bending
Ω : Load frequency

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